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INVESTIGATION OF THE PROCESS OF SIMPLE WAVE INVERSION IN AN ANISOTROPIC PLASMA

M. S. Ruderman

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16. Abstract

The inversion path of simple waves in anisotropic plasma is determined on the basis of the Chew, Goldberg, and Low equations. The validity is examined of applying the linear theory to the question of heating of the outer solar corona by rapid magnetosonic waves.

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INVESTIGATION OF THE PROCESS OF SIMPLE WAVE INVERSION IN AN ANISOTROPIC PLASMA

M. S. Ruderman

Introduction /3*

Many problems in space and laboratory physics lead to the necessity of investigating movements of plasma for which the characteristic parameter variation length is small in comparison to the mean free path of the charged particles but large in comparison to the Larmor radius.

Chew, Goldberg and Low [1] have shown that in this case the behavior of the plasma can be described by a system of magnetohydrodynamic equations using anisotropic pressure.

On the basis of these equations a study was made in [2] of simple waves, the example being taken of nonlinear waves and it being assumed that the longitudinal and transverse pressures ($p_{\parallel}$ and $p_{\perp}$) are small in comparison to magnetic pressure $B^2/8\pi$, in which $\vec{B}$ is the vector of magnetic field induction, and in [3] investigation is made of simple waves with no restrictions imposed on the hydrodynamic parameters.

The process of inversion of simple waves in the Chew, Goldberg, and Low (CGL) approximation is investigated in the present paper.

§1. Simple Waves in an Anisotropic Plasma /4

The system of magnetohydrodynamic equations in the CGL approximation is of the form (see, for example, [1]):

*Numbers in the margin indicate pagination in the foreign text.

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\rho\vec{v}=0;\quad \rho\frac{d\vec{v}}{dt}=-\operatorname{div}\overset{\leftrightarrow}{P}+\frac{1}{4\pi}\operatorname{rot}\vec{B}\times\vec{B};$$
$$\frac{\partial\vec{B}}{\partial t}=\operatorname{rot}[\vec{v}\times\vec{B}];\qquad \operatorname{div}\vec{B}=0;$$
$$\frac{d}{dt}\left(\frac{p_{\parallel}B^2}{\rho^3}\right)=0;\quad \frac{d}{dt}\left(\frac{p_{\perp}}{\rho B}\right)=0;\tag{1.1}$$
$$\operatorname{div}\overset{\leftrightarrow}{P}=\nabla p_{\perp}+(p_{\parallel}-p_{\perp})(\vec{b}\nabla)\vec{b}+\vec{b}\operatorname{div}(p_{\parallel}-p_{\perp})\vec{b}.$$

Here ρ is the density, $\vec{v}$ the mean velocity, $p_{\perp}$, $p_{\parallel}$ are the transfers and longitudinal pressures, $\overset{\leftrightarrow}{P}$ equals the pressure tensor, $\vec{B}$ is the magnetic field induction vector, and $\vec{b}=\vec{B}/B$ is the unit vector along the magnetic field.

Simple wave or Riemann wave is the term applied to a one-dimensional flow of gas in which all the hydrodynamic parameters depend on $\varphi(x, t)$, in which t is time and x is the space coordinate.

φ is an arbitrary function. We will assume that $\varphi \equiv \rho$. We introduce the phase velocity of wave motion relative to the gas in accordance with the formula

$$\alpha=\lambda-u,\tag{1.2}$$

in which λ is the phase velocity of wave motion and u is the velocity of gas motion along the x axis. It has been shown in [3] that α^2 may assume 4 values, of which we will consider only 2, those corresponding to rapid and slow Riemann waves. We designate these values as α_+^2 and α_-^2 respectively.

There is derived in [3] a system of equations ascribing the variation in parameters as a function of ρ in rapid and slow simple waves:

$$\frac{dB_y^2}{d\rho}=\frac{\ell B^2}{\rho}\cdot\frac{A_{+,-}}{c};\qquad \frac{du}{d\rho}=\frac{\alpha_{+,-}}{\rho};$$
$$\frac{p_{\perp}}{\rho B}=const;\qquad \frac{p_{\parallel}B^2}{\rho^3}=const;$$
$$\frac{dv}{d\rho}=\frac{B_x}{\rho^2\alpha_{+,-}B_y}\{(1-\ell)(3p_{\parallel}-p_{\perp})+[p_{\parallel}-p_{\perp}-(4p_{\parallel}-p_{\perp})(1-\ell)]\frac{A_{+,-}}{c}\};\tag{1.3}$$
$$B_x=const;$$
$$\alpha^2_{+,-}=\frac{p_{\perp}}{\rho}+\frac{B^2}{8\pi\rho}+\frac{1}{2\rho}(2p_{\parallel}-p_{\perp})\ell\pm\{[\frac{p_{\perp}}{\rho}+\frac{B^2}{8\pi\rho}+\frac{1}{2\rho}(2p_{\parallel}-p_{\perp})\ell]^2+$$
$$+[(\frac{p_{\perp}}{\rho})^2\ell(1-\ell)-\frac{3p_{\parallel}p_{\perp}}{\rho}\ell(2-\ell)+3(\frac{p_{\parallel}}{\rho}\ell)^2-\frac{3p_{\parallel}}{\rho}\ell\frac{B^2}{4\pi\rho}]\}^{1/2}.$$

Here Bx, By are the components of the magnetic field strength vector along the x and y axes, and v is the velocity of gas motion along the y axis. We additionally introduce the notation

$$\ell = \frac{B_x^2}{B^2}; \quad B^2 = B_x^2 + B_y^2; \quad p_m = p_\perp + \frac{B^2}{4\pi}; \quad A_{+,-} = \rho a_{+,-}^2 - (1-\ell)p_\perp - 3\ell p_\parallel; \quad c = p_m - \ell(4p_\parallel - p_\perp). \tag{1.4}$$

A qualitative study is made in [3] of the field of integral curves for rapid and slow waves in the ρ, B_y^2 plane. The field of integral curves for rapid waves is illustrated in Figure 1, and that for slow waves in Figure 2.

In both figures curve 1 is defined by the equation

$$p_\parallel = p_m . \tag{1.5}$$

to the right of curve 1 there is a region of serpentine instability defined by statement of a quality $p_\parallel > p_m$. The values of ρ^* and ρ^{**} are determined by the equation

$$p_\perp^* - 4p_\parallel^* + \frac{B_x^2}{4\pi} = 0; \quad p_m^{**} - 4p_\parallel^{**} + p_\perp^{**} = 0 . \tag{1.6}$$

Superscripts (*) and (**) denote that the values are taken at $B_y^2 = 0$ and $\rho = \rho^*$ /6
and $\rho = \rho^{**}$ respectively.

For slow simple waves along an integral curve emerging from the ρ axis to the left of ρ^{**}, $\alpha B_y^2/\alpha\rho < 0$ everywhere. If the integral curve emerges from the ρ axis to the right of ρ^{**}, then initially $\alpha B_y^2/\alpha\rho > 0$, this derivative subsequently becoming infinite, after which $\alpha B_y^2/\alpha\rho < 0$.

§2. Derivation of Equations for Inversion Time and Path

It is convenient for purposes of subsequent discussion to introduce the dimensionless variables:

$$\begin{aligned} &\beta = 8\pi p_\parallel / B_0^2, \quad \Delta p/p = (p_\parallel - p_\perp)/p_\parallel, \\ &\rho' = \rho/\rho_0, \quad a'^2_{+,-} = a^2_{+,-}\rho_0/B_0^2, \quad u' = u(\rho_0/B_0^2)^{1/2}, \\ &\tau = t(B_0^2/\rho_0 L^2), \quad \xi = x/L \end{aligned} \tag{2.1}$$

here ρ_0 is the characteristic density, L the characteristic dimension, and B_0 the magnetic induction at $\rho = \rho_0$.

Making use of (2.1) we reduce equations (1.3) and relations (1.4) to dimensionless form. As the result we obtain:

$$\frac{d(1/\ell)}{d\rho'} = \frac{2}{\ell\rho'}\frac{A'_{+,-}}{C'}\,; \quad \frac{du'}{d\rho'} = \frac{a'_{+,-}}{\rho'}\,;$$
$$\frac{\beta}{\rho'}\left(1-\frac{\Delta p}{p}\right)\ell^{1/2} = const\,; \quad \frac{\beta}{\ell\rho'^3} = const\,. \tag{2.2}$$

$$A'_{+,-} = \rho' a'^2_{+,-} - \frac{\beta}{8\pi}\left[(1+2\ell)-(1-\ell)\frac{\Delta p}{p}\right];$$
$$C' = \frac{\ell_c}{4\pi\ell} + \frac{\beta}{8\pi}\left[1-3\ell\ell_o-(1+\ell\ell_o)\frac{\Delta p}{p}\right]; \quad \ell_o = \frac{B_x^2}{B_o^2},$$
$$a'_{+,-} = \frac{1}{8\pi\rho'}\left\{\left[\left(1-\frac{\Delta p}{p}\right)\beta + \frac{\ell}{\ell_o} + \frac{\beta}{2}\ell\left(1+\frac{\Delta p}{p}\right)\right] \pm\right.$$
$$\pm\left[\left[\left(1-\frac{\Delta p}{p}\right)\beta + \frac{\ell_o}{\ell} + \frac{\beta}{2}\ell\left(1+\frac{\Delta p}{p}\right)\right]^2 + \beta^2\left[\ell\left(1-\frac{\Delta p}{p}\right)(1-\ell) - \right.\right.$$
$$\left.\left.\left. -3\left(1-\frac{\Delta p}{p}\right)\ell(2-\ell) + 3\ell^2 - \frac{6}{3}\ell_o\right]\right]^{1/2}\right\}. \tag{2.3}$$

We shall henceforth omit the primes accompanying the dimensionless values. /7

In order to find $\rho(\tau, \xi)$ it is necessary to solve the equation

$$\frac{\partial\rho}{\partial\tau} + \rho\frac{\partial u}{\partial\xi} + u\frac{\partial\rho}{\partial\xi} = 0,$$

in which

$$\frac{\partial u}{\partial\xi} = \frac{du}{d\rho}\frac{\partial\rho}{\partial\xi} = \frac{a_{+,-}}{\rho}\cdot\frac{\partial\rho}{\partial\xi}\,.$$

Hence for ρ we have the equation:

$$\frac{\partial\rho}{\partial\tau} + (u + a_{+,-})\frac{\partial\rho}{\partial\xi} = 0\,. \tag{2.4}$$

The equation of characteristics and the characteristic relation for equation (2.5) are of the form

$$\frac{d\xi}{d\tau} = a_{+,-} + u\,; \qquad \rho = const, \tag{2.5}$$

from which it follows that the characteristics are straight lines. Inversion occurs at the time when 2 infinitely close characteristics intersect for the first time.

Let

$$\rho\Big|_{\tau=0} = \begin{cases} f(0) & \xi < 0 \\ f(\xi) & 0 \leq \xi \leq 1 \\ 1 & \xi > 1\,, \end{cases} \tag{2.6}$$

with $\alpha f/\alpha\xi \leq 0$ when $0 \leq \xi \leq 1$, $f(1) = 1$. In plane τ, ξ the characteristics emerging from the ξ axis to the right of 1 and to the left of 0 will be parallel.

Only the characteristics emerging from segment [0, 1] can change in slope. In particular, if $\partial\lambda/\partial\rho > 0$, the angle between the characteristics and the ξ axis will decrease with movement from 0 to 1.

Let $0 \leqslant \xi_0 \leqslant 1$. Let us consider the characteristics emerging from points ξ_0 and $\xi_0 + \Delta\xi$. The equations for them are written in the form

$$\xi = \lambda(f(\xi_0))\cdot\tau + \xi_0;$$
$$\xi = \lambda(f(\xi_0+\Delta\xi))\cdot\tau + \xi_0 + \Delta\xi,$$

in which $\lambda = \alpha + u$. By finding the point of intersection of these 2 lines and passing to the limit at $\Delta\xi \to 0$, we learn that infinitely close characteristics intersect at the point /8

$$\tau = \left[-\frac{d\lambda}{d\rho}\cdot\frac{df}{d\xi}\right]^{-1}_{\xi=\xi_0}.$$

Then the inversion time is defined by the formula

$$\tau_{inv} = \min_{0\leq\xi\leq 1}\left[-\frac{d\lambda}{d\rho}\bigg|_{\rho=f(\xi)}\cdot\frac{df}{d\xi}\right]^{-1}. \tag{2.7}$$

If the inversion path is defined as the distance covered by the base of the profile of (2.6) in τ_{inv} with $u(1) = 0$ (i.e., over a stationary medium), the inversion path is then recorded in the form

$$\xi_{inv} = \tau_{inv}\cdot\alpha(1). \tag{2.8}$$

§3. Determination of Inversion Path in Certain Extreme Cases for Rapid Waves

1. $\beta \ll 1$, $\beta\left(1-\frac{\Delta p}{p}\right) \ll 1$.

These statements of inequality denote that $p_{\parallel}$ and $p_{\perp}$ are much smaller than the magnetic pressure.

Equations (2.2) assume the form

$$\frac{d(1/\ell)}{d\rho} = \frac{2}{\ell\rho}; \quad \frac{du}{d\rho} = \frac{1}{\rho\sqrt{4\pi\rho}}\cdot\left(\frac{\ell_0}{\ell}\right)^{1/2};$$
$$\frac{\beta}{\rho}\left(1-\frac{\Delta p}{p}\right)\ell^{1/2} = \beta_0\left(1-\frac{\Delta p_0}{p_0}\right)\ell^{1/2}; \quad \frac{\beta}{\ell\rho^3} = \frac{\beta_0}{\ell_0}. \tag{3.1}$$

From the first equation we obtain $l = l_0\rho^{-2}$. It is then easy to find that

$$\frac{d\lambda}{d\rho} = \frac{d\alpha_+}{d\rho} + \frac{du}{d\rho} = \frac{3}{2}\frac{1}{\sqrt{4\pi\rho}}$$

By use of this result we obtain the following expression for the inversion path /9

$$\xi_{inv} = \frac{2}{3}\min_{0\le\xi\le1}\left(-\sqrt{\frac{1}{f(\xi)}}\cdot\frac{df}{d\xi}\right)^{-1}. \tag{3.2}$$

Let

$$f(\xi) = \alpha(1-\xi^2)+1, \quad \alpha>0. \tag{3.3}$$

(The graph of this function is shown in Figure 3.) $\xi_{inv} = 1/3\,\alpha$, the profile being inverted at the base, that is, when $\xi = 1$.

2. $l \ll 1$. Equations (2.2) assume the form:

$$\begin{aligned} &\frac{d(1/\ell)}{d\rho} = \frac{2}{\ell\rho}; \qquad \frac{du}{d\rho} = \frac{\alpha_+}{\rho};\\ &\frac{\beta}{\rho}\left(1-\frac{\Delta p}{p}\right)\ell^{1/2} = \beta_0\left(1-\frac{\Delta p}{p}\right)\ell_0^{1/2}; \quad \frac{\beta}{\ell\rho^3} = \frac{\beta_0}{\ell_0}; \end{aligned} \tag{3.4}$$

with

$$\alpha_+^2 = \frac{1}{4\pi\rho}\left[\left(1-\frac{\Delta p}{p}\right)\cdot\beta + \ell_0/\ell\right].$$

From the first equation of system (3.4) we obtain $l = l_0\rho^{-2}$, after which it is easy to find the following expression for the inversion path:

$$\xi_{inv} = \frac{2}{3}\min_{0\le\xi\le1}\left(-\sqrt{\frac{1}{f(\xi)}}\cdot\frac{df}{d\xi}\right)^{-1}. \tag{3.5}$$

Adopting $f(\xi)$ in the form of (3.3) we obtain $\xi_{inv} = 1/3\,\alpha$.

§4. Determination of Inversion Path in Certain Extreme Cases for Slow Waves

1. $\beta \ll 1$, $\beta\left(1-\frac{\Delta p}{p}\right) \ll 1$.

Equations (2.2) assume the form (see [3]):

$$\begin{aligned} &\frac{d(1/\ell)}{d\rho} = -(1-\ell)\frac{\beta}{\rho}\left(1-\frac{\Delta p}{p}\right); \quad \frac{du}{d\rho} = \frac{\alpha_-}{\rho};\\ &\frac{d}{d\rho}\left[\beta\left(1-\frac{\Delta p}{p}\right)\right] = \frac{\beta}{\rho}\left(1-\frac{\Delta p}{p}\right); \quad \frac{d\beta}{d\rho} = \frac{3\beta}{\rho}. \end{aligned} \tag{3.6}$$

We obtain the following expression for α_-^2

$$\alpha_-^2 = \beta\ell/8\pi\rho$$

Integrating system (3.6) we obtain /10

$$\begin{aligned} &\beta\left(1-\frac{\Delta p}{p}\right) = \beta_0\left(1-\frac{\Delta p_0}{p_0}\right)\rho; \quad \beta = \beta_0\rho^3;\\ &\rho = 1 - \frac{1}{\beta_0\left(1-\frac{\Delta p_0}{p_0}\right)}\left[\left(1/\ell - 1/\ell_0\right) + \ln\frac{\ell_0(1-\ell)}{\ell(1-\ell_0)}\right]. \end{aligned} \tag{3.7}$$

It can be readily seen that ρ is a monotonically decreasing function of $1/l$. Then $1/l$ is in its turn a monotonically decreasing function of ρ. We introduce the notation

$$1/\ell = F(\rho).$$

It is then easy to obtain the following equations

$$\alpha_- = \sqrt{\frac{3\beta_0}{8\pi F(\rho)}}\cdot\rho; \qquad \frac{du}{d\rho} = \sqrt{\frac{3\beta_0}{3\pi F(\rho)}};$$
$$\frac{d\lambda}{d\rho} = \sqrt{\frac{3\beta_0}{2\pi F(\rho)}}\cdot\left[1 + \frac{\beta_0\left(1-\frac{\Delta p_0}{p_0}\right)(F(\rho)-1)\cdot\rho}{[F(\rho)]^2}\right]. \tag{3.8}$$

The last term in (3.8) may be disregarded because β_0 is so small, and we then obtain

$$\xi_{inv} = \frac{\sqrt{\ell_0}}{2}\min_{0\le\xi\le1}\left[-\frac{1}{\sqrt{F(f(\xi))}}\cdot\frac{df}{d\xi}\right]^{-1}. \tag{3.9}$$

Let us take $f(\xi)$ in the form of (3.3). By introducing the notation $Z = 1 - \xi^2$, we obtain the following formula for the inversion path

$$\xi_{inv} = \frac{\sqrt{\ell_0}}{4\alpha}\min_{0\le Z\le1}\sqrt{\frac{F(\alpha Z+1)}{1-Z}}. \tag{3.9'}$$

The following equation can easily be obtained by use of (3.6):

$$\frac{d}{dZ}\left[\frac{F(\alpha Z+1)}{1-Z}\right] = \frac{1}{(1-Z)^2}\left\{\left[-(1-\ell)\frac{\alpha\beta}{\rho}\left(1-\frac{\Delta p}{p}\right)\right]\Big|_{\rho=\alpha Z+1} + F(\alpha Z+1)\right\}.$$

The first term in the braces may be disregarded because β is so small, provided only that α is not too large. Then $F(\alpha Z + 1)/(1 - Z)$ is a monotonically increasing function and the minimum in (3.9') is reached when $Z = 0$. Thus we /11 obtain for the inversion path

$$\xi_{inv} = 1/4\alpha. \tag{3.10}$$

2. $l \ll 1$. Equations (2.2) assume the form

$$\frac{d(1/\ell)}{d\rho} = -\frac{2\beta\left(1-\frac{\Delta p}{p}\right)}{\rho\left[2\ell_0 + \beta\ell\left(1-\frac{\Delta p}{p}\right)\right]}; \qquad \frac{du}{d\rho} = \frac{\alpha_-}{\rho};$$
$$\beta\left(1-\frac{\Delta p}{p}\right) = \beta_0\left(1-\frac{\Delta p_0}{p_0}\right)\left(\frac{\ell_0}{\ell}\right)^{1/2}\rho; \qquad \beta = \beta_0\frac{\ell}{\ell_0}\rho^3. \tag{3.11}$$

We have the following expression for α_-^2

$$\alpha_-^2 = \frac{3\beta\ell}{8\pi\rho}\left\{1 - \frac{(1-\Delta p/p)^2\beta}{6[(1-\Delta p/p)\beta + \ell_0/\ell]}\right\}.$$

By integrating (3.11) we obtain

$$\left(\frac{\ell_0}{\ell}\right)^{1/2} = \frac{\beta_0\left(1-\frac{\Delta p_0}{p_0}\right)}{2}\left\{-\rho+\sqrt{\rho^2+\frac{4[1+\beta_0(1-\Delta p_0/p_0)]}{\beta_0^2(1-\Delta p_0/p_0)^2}}\right\}. \qquad (3.12)$$

It can readily be seen that $(l_0/l)^{1/2}$ decreases monotonically with ρ increasing from 1 to $+\infty$ and $(l_0/l)^{1/2} \to 0$ when $\rho \to \infty$.

For the purposes of the following discussion we shall assume that $\beta_0 \gg 1$, $\beta_0(1 - \Delta p_0/p_0 \gg 1$. Then we obtain the following equations from (3.11) and (3.12):

$$\ell = \ell_0\rho^2; \quad \beta\left(1-\frac{\Delta p}{p}\right) = \beta_0\left(1-\frac{\Delta p_0}{p_0}\right); \quad \beta = \beta_0\rho^5, \qquad (3.13)$$

$$\frac{d\lambda}{d\rho} = \frac{1}{\rho}\frac{d\rho\alpha_-}{d\rho} = \left(\frac{3\ell_0\beta_0}{8\pi}\right)^{1/2}\left[4\rho^5-\frac{1}{4}\left(1-\frac{\Delta p_0}{p_0}\right)\right]\cdot\left[\rho^6-\frac{\rho}{6}\left(1-\frac{\Delta p_0}{p_0}\right)\right]^{-1/2}, \qquad (3.14)$$

$$\alpha_-(1) = \left(\frac{3\ell_0\beta_0}{8\pi}\right)^{1/2}\left(\frac{5}{6}+\frac{1}{6}\frac{\Delta p_0}{p_0}\right)^{1/2}. \qquad (3.15)$$

It follows from (3.15) that $\alpha_-^2 > 0$ when $\Delta p_0/p_0 > -5$. We shall henceforth assume this condition to have been fulfilled.

f Let us take f(ξ) in the form of (3.3). We then obtain the following expression for the inversion path:

$$\xi_{\text{inv}} = \frac{1}{2\sqrt{\alpha}}\left(\frac{5}{6}+\frac{1}{6}\frac{\Delta p_0}{p_0}\right)^{1/2}\min_{1\le\rho\le 1+\alpha}\varphi(\rho), \qquad (3.16)$$ /12

in which

$$\varphi(\rho) = \frac{\left[\rho^6-\frac{\rho}{6}\left(1-\frac{\Delta p_0}{p_0}\right)\right]^{1/2}}{\sqrt{1+\alpha-\rho}\left[4\rho^5-\frac{1}{4}\left(1-\frac{\Delta p_0}{p_0}\right)\right]}.$$

We easily obtain

$$\frac{d\varphi}{d\rho} = \frac{\varphi_1(\rho)}{(1+\alpha-\rho)\left[4\rho^5-\frac{1}{4}\left(1-\frac{\Delta p_0}{p_0}\right)\right]\left[\rho^6-\frac{\rho}{6}\left(1-\frac{\Delta p_0}{p_0}\right)\right]^{1/2}}, \qquad (3.17)$$

in which

$$\varphi_1(\rho) = 10\rho^{11} - 8(1+\alpha)\rho^{10} - \frac{65}{24}\rho^6\left(1-\frac{\Delta p_0}{p_0}\right) + \frac{9}{4}(1+\alpha)\rho^5\left(1-\frac{\Delta p_0}{p_0}\right) + \frac{1+\alpha}{48}\left(1-\frac{\Delta p_0}{p_0}\right)^2.$$

function $\varphi_1(\rho)$ is a quadratic trinomial relative to $(1 - \Delta p_0/p_0)$. Let us calculate its discriminant $D(\rho)$. We have

$$\mathcal{D}(\rho) = \frac{25\rho^{10}}{576}\left[169\rho^2 - 300\rho(1+\alpha) + 128(1+\alpha)^2\right]. \qquad (3.18)$$

$D(\rho) < 0$ when $0.707(1 + \alpha) < \rho < 1.06(1 + \alpha)$. If $\alpha < 0.414$, then $0.707\ (1 + \alpha) < 1$. In this case $\alpha\varphi/\alpha\rho > 0$ on segment $[1,\ 1 + \alpha]$. Then $\varphi(\rho)$ assumes the minimum value when $\rho = 1$. Thus for the inversion path we obtain

$$\xi_{inv} = \frac{5 + \Delta p_0/p_0}{36\alpha} \qquad \text{when } \alpha < 0.414 . \tag{3.19}$$

When $\Delta p_0/p_0 = -3$, $\alpha = 0.3$, we have $\xi_{inv} = 0.185$, i.e., the wave is inverted after approximately one-fifth of its length L has passed.

It must be noted that when $\beta_0 \gg 1$ the condition of occurrence of serpentine instability is written in the form $\beta > \beta(1 - \Delta p/p)$ or $\Delta p/p > 0$. But it follows from (3.11) that

$$\Delta p/p = 1 - (1 - \Delta p_0/p_0)\rho^{-5} .$$

Thus we find that on change in ρ from 1 to $1 + \alpha$ the condition of absence of serpentine instability is expressed by the statement of inequality

$$\frac{\Delta p_0}{p_0} < -(1+\alpha)^5 + 1 . \tag{3.20}$$

This condition will be used subsequently in discussion of the numerical results.

§5. Determination of the Inversion Path by a Numerical Method /13

In the general case it is not possible to investigate expression (2.7) analytically and determine the inversion path. Hence a numerical method is utilized.

Equation (2.2) is integrated and formula

$$\alpha_1 \frac{d\lambda}{d\rho} = \alpha_2 \frac{A_{+,-}}{c} + \alpha_3 . \tag{4.1}$$

derived in [3] is used for numerical determination of the inversion path. Coefficients a_1, a_2, a_3 are given in [3] and are not reproduced here because of being too unwieldy. ξ_{inv} is determined by means of formulas (2.7) and (2.8). $f(\xi)$ was in this case selected in the form of (3.3) (see Figure 3).

ξ_{inv} as a function of $1/l_0$ has been calculated for rapid waves, it being assumed that $\alpha = 1$. Five cases have been considered:

$\beta_0 = 0.04$; $\Delta p/p_0 = 0$; curve 1;
$\beta_0 = 0.4$; $\Delta p/p_0 = 0$; curve 2;
$\beta_0 = 4$; $\Delta p/p_0 = 0$; curve 3;
$\beta_0 = 1$; $\Delta p/p_0 = 0.9$; curve 4;
$\beta_0 = 0.0571$; $\Delta p/p_0 = -9$; curve 5.

$1/l_o$ in this case ranges from 1 to 11. The results of the calculation are presented in Figure 4. It is to be noted that the 2nd, 4th and 5th cases are characterized by the fact that when $\rho = 1$ the value of the voltage tensor deviator is the same and its ratio to the magnetic pressure equals 0.4.

The inversion path as a function of β, with $\alpha = 0.03$, was determined for slow magnetosonic waves. In accordance with (3.20), at large values β the serpentine instability does not occur at

$$\Delta p_o/p_o < -(1.3)^5 + 1 \approx -2.713.$$

$\Delta p_o/p_o = -3$ was selected for the numerical calculations. The next difficulty is /14
caused by the fact that at $1/l_o \sim 1$, over a sufficiently wide range of β_o values we encounter an integral curve $B_y^2(\rho)$ such as does not permit increase in ρ by a factor of 1.3 (see Figure 2). In order to surmount this difficulty, it is assumed that $l = 0.01$. In this case the portions of the integral curves on which B_y^2 increase with increase in ρ are displaced far to the right along the ρ axis.

In the numerical calculations β ranged from 0.01 to 5. The results of the calculations are presented in Figure 5.

It is to be seen from Figure 4 that when $l_o \to 0$, $\xi_{inv} \to 1/3$, with $\beta \ll 1$, ξ_{inv} is near 1/3 at any value l. This is in close agreement with the results of §3. Unfortunately, it is not possible to devise a logical physical explanation for the behavior of the curves at $1/l_o \sim 1$. However, it must be remembered that the CGL equations are valid only for motion perpendicular to the magnetic field. In addition, cold plasma is described by equations which are equations of discontinuity and conservation of momentum. For this reason it is precisely the limiting cases discussed in §3 which are of the greatest interest.

It is to be seen from Figure 5 that when $\beta \to \infty$, $\xi_{inv} \to 0.185$, and when $\beta \to 0$, $\xi_{inv} \to 0.833$, this agreeing with the results of §4.

§6. Effect of Wave Inversion on Solar Wind Heating

The following mechanism of solar wind heating is proposed in [4]. It is assumed that there emerges from the base of the external corona an energy flux of the order of 5×10^{26} erg/sec in the form of hydromagnetic waves. The lines of force of the magnetic field are considered to be in the form of spirals, in

keeping with the models discussed in [5]. In particular, it is assumed that the magnetic field of the Sun is almost radial up to distances of the order of 10 $R_\odot$, where $R_\odot$ is the radius of the Sun. It is believed that the spectrum of the waves emerging from the base of the external corona is isotropic. As has been demonstrated in [6], slow magnetosonic waves are damped at a distance of the order of several wavelengths. It is also demonstrated here that the rapid /15
waves are also rather intensely damped, provided that they are not propagated along or across the magnetic field. It is shown in [4] that the wave vector tends to become radial when waves are propagated from the Sun through interplanetary plasma. Owing to these two processes (damping and the tendency of waves toward radial propagation), approximately 1% of the wave energy remains at a distance of the order of several solar radiuses, this energy being propagated in the form of radial waves. These waves subsequently enter a region in which the magnetic field ceases to be radial and begins to damp, increasing the temperature of the ions along the magnetic field, as has been demonstrated in [7].

It is demonstrated below that the inversion path of the rapid magnetosonic waves referred to in the foregoing is much shorter than the solar radius. Consequently, without being inverted they cannot reach the regions in which the magnetic field ceases to be radial and the waves are greatly damped. When a wave is inverted its profile becomes steeper, this leading to the occurrence of shortwave harmonics in expansion of the wave into a Fourier series. A time comes when the wavelengths of these harmonics become comparable to the Larmor radius of the ions, and the conclusions drawn in [6] consequently do not apply to them. It may be found, in particular, that they are greatly damped on being propagated along the magnetic field. Hence it is necessary to allow for nonlinear effects in wave propagation in the theory advanced in [4]. Let us determine the inversion path of rapid radially propagated magnetosonic waves at a distance $r \simeq 10\ R_\odot$. The following solar wind parameters at $r \simeq 10\ R_\odot$ are given in [4]: ion density $N \simeq 10^4 \text{cm}^{-3}$, temperature $T \simeq 10^{6}$°K, magnetic induction $B \simeq 10^{-2}$ gauss. In addition, it is pointed out in [4] that on change in r from 2 $R_\odot$ to 10 $R_\odot$ N decreases as r^{-3}. It is then easy to obtain the following approximate expression for N:

$$N = 10^7 (R_\odot / r)^3 . \qquad (4.1)$$ /16

Say that at distance r_{one} N changes by 1%. The following approximate formula can be obtained for r_{one}:

$$r_{one} \simeq 0.0033 r. \tag{4.2}$$

In particular, when $r \simeq 10\ R_\odot$, $r_{one} \simeq 2 \cdot 10^4$ km. The solar plasma may clearly be considered to be homogeneous at a distance of the order of r_{one}.

Let l_{ll} be the mean free path of the electrons, and r_{li} the Larmor radius of the ions. Employing the formula for a fully ionized Coulomb gas, we obtain $l_{ll} \simeq 10^5$ km. For the Larmor ion radius we have the value $r_{li} \simeq 1$ km. The CGL equations are then applicable to waves length L of which satisfies the following statement of inequality:

$$1 \text{ km} \ll L \ll 10^5 \text{ km}. \tag{4.3}$$

However, we are considering waves coming from the base of the external corona. When $r \simeq 2\ R_\odot$ we have the following values of l_{ll} and r_{li}: $l_{ll} \simeq 10^3$ km, $r_{li} \simeq 100$ m. Hence in the range from 2 $R_\odot$ to 10 $R_\odot$ the CGL equations are valid for waves having a length

$$L \simeq 10 \div 100 \text{ km}. \tag{4.4}$$

When $r \simeq 10\ R_\odot$ the temperature in the solar wind is isotropic and the temperatures of the ions and electrons are equal. We obtain the following values for the thermal and magnetic pressures: $p_0 \simeq 3 \cdot 10^{-6}$ dyn/cm^2, $B_0^2/8\pi \simeq 4 \cdot 10^{-6}$ dyn/cm^2. The speed of the solar wind $u \simeq 10^7$ cm/sec = 100 km/sec. We obtain the following phase velocity for waves propagated along the magnetic field:

$$a_+ = \left(\frac{3p_0}{\rho_0} \right)^{1/2}. \tag{4.5}$$

By means of primes we will designate small disturbances of the corresponding values. For a rapid wave we obtain $\vec{B}' = 0$, $v' = 0$ and thus $\vec{E}'_\perp = 0$. Since in the CGL $\vec{E}_\parallel = 0$, we obtain $\vec{E}' = 0$. Hence ρ', u', $p'_\perp$, and $p'_\parallel$ undergo change in a rapid wave propagated along the magnetic field. The mean energy of the wave may be estimated as follows: /17

$$\langle W \rangle_{wave} \simeq \frac{1}{2} \rho_0 \langle u'^2 \rangle. \tag{4.6}$$

The symbol $\langle\ \rangle$ denotes averaging over space. Velocity and density disturbances are related as follows in a rapid wave:

$$u' = \frac{a_+}{\rho_0}\rho' . \tag{4.7}$$

Say that all energy is propagated in the form of waves of length L and with a ratio of amplitude of density disturbance to undisturbed density equalling α. Then

$$\langle W \rangle_{wave} \simeq \frac{1}{4}\alpha^2 \rho_0 a_+^2 . \tag{4.8}$$

The waves transfer through a spherical surface of radius r energy equalling

$$W = 4\pi r^2 (a_+ + u) \cdot \frac{1}{4}\alpha^2 \rho_0 a_+^2 . \tag{4.9}$$

Substituting in (4.8) $r = 10\ R_\odot$ and the values of a_+, u_0, and ρ_0 corresponding to the distance from the Sun, we obtain for W

$$W = 5 \cdot 10^{26} \cdot \alpha^2 \text{ erg/sec.} \tag{4.10}$$

But on the other hand, $W = 5 \cdot 10^{24}$ erg/sec (see [4]). Thus, $\alpha \simeq 0.1$. It is shown in the appendix that a simple wave propagated along the magnetic field is inverted after traveling a path of the order of L/α. In our case the inversion path will thus be of the order of 10 L. It follows from (4.4) that the inversion will take place at a distance not exceeding 1,000 km, this being smaller than r_{one} and even smaller than $R_\odot$.

It is to be noted that the conclusions drawn in this section are based on application of the CGL equations to motion of a plasma along a field, and there is little justification for this. /18

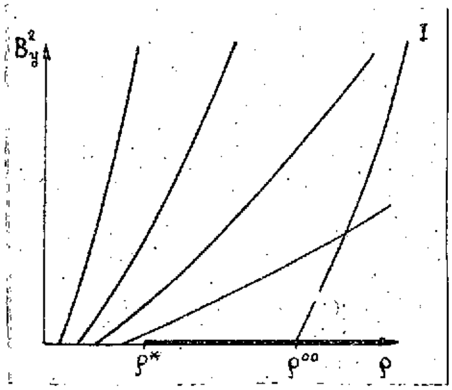

Figure 1. Illustrating Trace of Integral Curves for Rapid Simple Waves. A region of serpentine instability is situated to the right of curve 1.

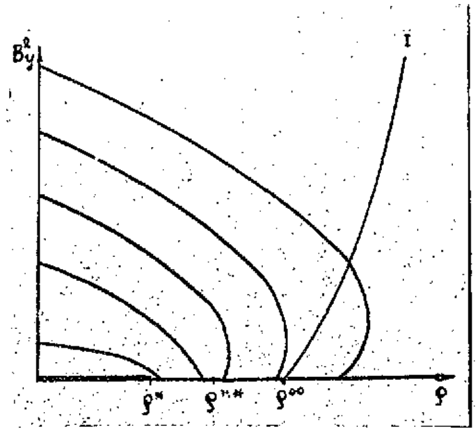

Figure 2. Illustrating Trace of Integral Curves for Slow Simple Waves. A region of serpentine instability is situated to the right of curve 1. /21

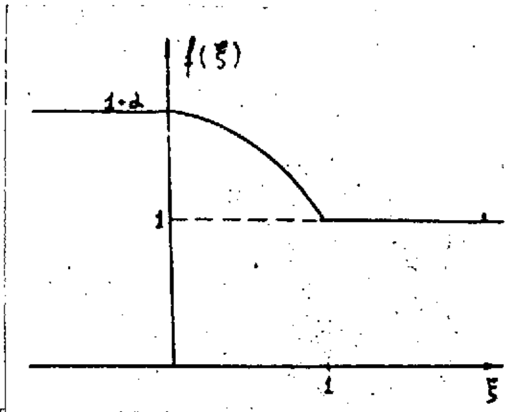

Figure 3. Solid-Line Curve: Graph of Function. /22

$$f(\xi)=\begin{cases} 1+\alpha & \text{for } \xi<0 \\ 1+\alpha(1-\xi) & \text{for } 0\leq\xi\leq 1 \\ 1 & \text{for } \xi>1 \end{cases}$$

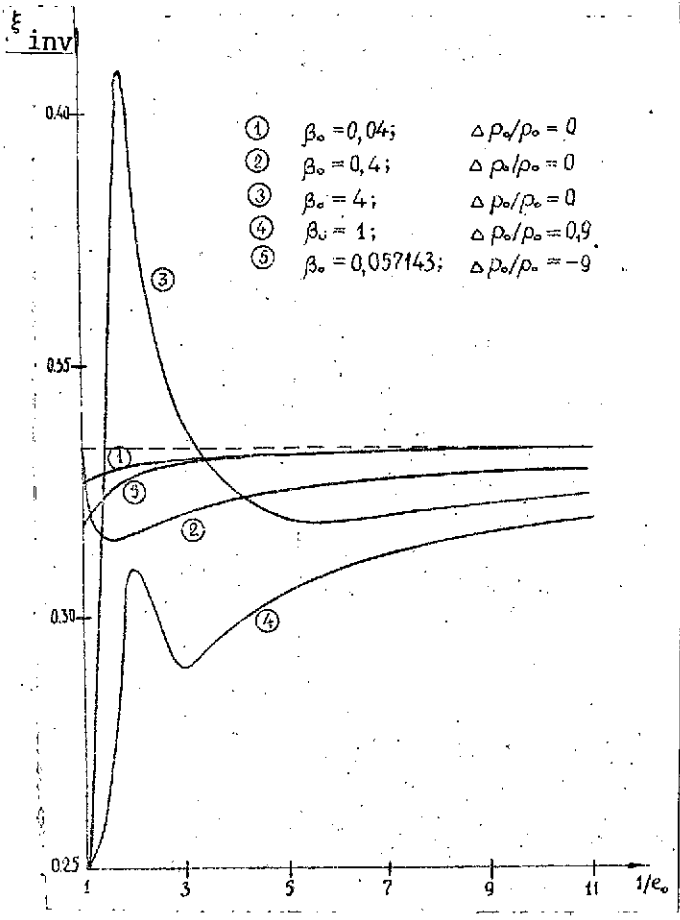

Figure 4. Illustrating Inversion Path as a Function of Direction of Magnetic Field in an Undisturbed Flux for Rapid Simple Waves. The broken line corresponds to ξ_{inv} = = 1/3. /23

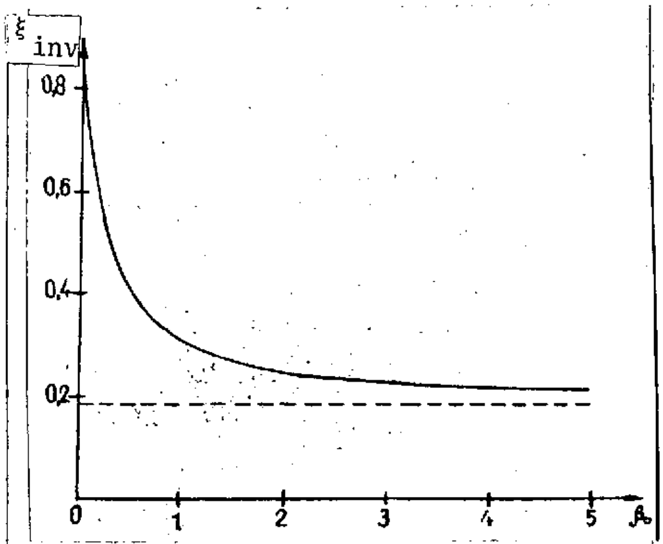

Figure 5. Illustrating Inversion Path as a Function of Ratio of Pressure Tensor Component Parallel to Magnetic Field to Magnetic Pressure for Slow Simple Waves Propagated Virtually Athwart the Magnetic Field. The broken line corresponds to ξ_{inv} = = 0.185. /24

APPENDIX

Let us construct the precise solution of equations (2.2). We make use of the fact that when $l = A_+ = 0$ when $\rho > \rho^*$, $A_- = 0$ when $\rho < \rho^*$ (see Figures 1 and 2), in which ρ^* is defined by equation (1.5) may be rewritten as follows in dimensionless form:

$$4\beta_0 \rho^3 = \beta_0 \left(1 - \frac{\Delta p_0}{p_0}\right)\rho + 1 . \tag{1}$$

Let us consider two simple waves, one following closely behind the other along the magnetic field (i.e., $l \equiv 1$). In the first, slow, wave, ρ varies from 1 to ρ^*, and in the second from ρ^* to $1 + \alpha$. If $1 + \alpha < \rho^*$, it suffices to consider the first wave only. We determine the phase velocity relative to the medium at rest, α, as follows:

$$\alpha = \begin{Bmatrix} a_- & \rho < \rho^* \\ a_+ & \rho > \rho^* \end{Bmatrix} = \sqrt{\frac{3\beta_0}{8\pi}}\,\rho . \tag{2}$$

It can readily be seen that a solution such as this satisfies (2.2), since it converts the righthand and lefthand members of the equation for $1/l$ into identical zeroes. In this wave the longitudinal velocity and the pressure tensor components change with change in ρ. The transverse components of the magnetic field and velocity remain equal to zero.

On the basis of (2) it is easy to obtain

$$\frac{d\lambda}{d\rho} \equiv \frac{1}{\rho}\frac{d\rho a}{d\rho} = 2\sqrt{\frac{3\beta_0}{8\pi}} , \tag{3}$$

from which is derived the formula for a dimensionless inversion path

$$\xi_{inv} = \frac{1}{2}\left(\max_{0 \le \xi \le 1} |f'|\right)^{-1} . \tag{4}$$ /19

Taking $f(\xi)$ in the form of (3.3), we obtain for the dimensionless inversion path

$$\xi_{inv} = 1/4\alpha . \tag{5}$$

When $f(\xi) = 1 + \alpha \cos \pi\xi/2$, $\xi_{inv} = 1/\pi\alpha$.

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